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Learning Mathematics as a Language

Kathryn Shafer

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Learning Mathematics as a Language

(TITLE)

BY

Kathryn Shafer

THESIS

SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
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ABSTRACT

This paper explores the relationship between language and mathematics. It is a summary of research done over the last thirty years. Also included are personal observations which are not part of any controlled study. Since language is the vehicle for thought, mathematics educators and curriculum planners will benefit from a linguistic approach to mathematics education. Symbolic mathematics is similar to natural language in both its structure and its communicative nature. If the students are to internalize the notation, they must be the ones to give it meaning. A linguistic approach to mathematics education includes language development, verbalization of concepts, vocabulary development, and written work. The child learns language through a sequence of listening, speaking, reading, and writing. This sequence is inherent in problem solving. The true purpose of mathematics education is to equip the student with the ability to understand a problem, formulate a plan to solve it, carry out that plan, and be able to tell if the answer they get is reasonable. An approach to mathematics instruction that addresses the language of mathematics will help provide the student with this ability.

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With much love and gratitude for the support I have received through the years, I dedicate this thesis to my mother, Carol Gage.

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CHAPTER I

MATHEMATICS AS A LANGUAGE

Introduction

Why should we look at mathematics as a Language? The development of language, first in the spoken form and then in a written form, has long been a concern of educators. Children initially learn language with little formal guidance. Language is used to communicate thoughts. To communicate effectively, one must pass on knowledge through language. Mathematics is learned through participation in meaningful activity and is communicated through both oral and written language. Much of the written language of mathematics is symbolic. The mathematic symbols represent concepts much the same as the written symbols of a native language. J. Allen Hickerson (1959) cited the following similarities between language and arithmetic:

Language

1. Language symbols (words or sentences) represent things, actions, ideas, relationships, etc.

2. The meaning of language symbols derive from that which they represent.

3. Communication through language involves speaking, listening, writing or reading.

4. The study of language is separated into semantics, mechanics, and grammar. (a) *Semantics*: the origin

and meanings of words, i.e., what word symbols represent; (b) *mechanics* of manipulating and recognizing language symbols: pronunciation, enunciation, inflection (speaking); hearing and distinguishing spoken sounds (listening); penmanship, spelling, sentence structure, punctuation (writing); eye movements, letter and word recognition, word analysis, eye span (silent reading); pronunciation, enunciation, eye span, expression (oral reading); (c) *grammar*: etymology (the nature of words in sentences); the rules, principles, generalizations concerning the nature of the structure of the language symbolism.

<u>Arithmetic</u>

1. Arithmetic symbols (numerals and numbers with operational signs) represent things, actions, ideas, relationships, etc.

2. The meaning of arithmetic symbols derive from that which they represent.

3. Communication through arithmetic involves speaking, listening, writing or reading.

4. The study of arithmetic is separated into (a) the *meanings* of arithmetical symbols: what the symbols represent, i.e., the quantities of things and quantitative relations among things; (b) *mechanics* of manipulating and recognizing arithmetic symbols; pronunciation and enunciation of the vocabulary of arithmetic (speaking); hearing and distinguishing sounds of spoken arithmetic words (listening); formation of numbers, signs, and symbols, and structure of algorisms (writing); eye movements, recognition of single numerals and multidigit numbers, recognition of signs and symbols, eye span (reading); (c) *the nature of the structure of the number system*, i.e., number notation and the rules, principles, generalizations concerning the nature of the arithmetic symbolism. (p. 241)

Mathematics in general, which includes arithmetic, is often referred to as a universal language. The symbolism is fairly uniform throughout the world. Students learning mathematics feel that mathematics is similar to a foreign language. The precision and level of abstraction that is necessary is not part of their everyday language. Symbolic mathematics, language development, verbalization, vocabulary, and writing are all topics of major concern and will be addressed in this paper.

Mathematics Standards

The current curriculum guide for teachers of mathematics in the United States is the *Curriculum and Evaluation Standards for School Mathematics (1989)* developed by the National Council of Teachers of Mathematics (NCTM). The mathematics education community called for reform in the teaching of mathematics. The NCTM Standards effectively responds to this challenge by formulating a curriculum guide that is responsive to the needs of society. Learning is a lifelong process. Our society is advancing so quickly that the average worker will have to be flexible to remain employed. School mathematics should provide a dynamic form of literacy. "Problem solving--which includes the ways in which problems are represented, the meanings of the language of mathematics, and the ways in which one conjectures and reasons--must be central to schooling..." (p. 4).

The NCTM Standards articulate five general goals for all students:

1. that they learn to value mathematics,
2. that they become confident in their ability to do mathematics,
3. that they become mathematical problem solvers,
4. that they learn to communicate mathematically, and

5. that they learn to reason mathematically. (p. 4)

Goal number four specifically states:

> The development of a student's power to use mathematics involves learning the signs, symbols, and terms of mathematics. This is best accomplished in problem situations in which students have an opportunity to read, write, and discuss ideas in which the use of the language of mathematics becomes natural. As students communicate their ideas, they learn to clarify, refine, and consolidate their thinking. (p. 4)

The NCTM Standards recognize that "mathematics is more than a collection of concepts and skills to be mastered; it includes methods of investigating and reasoning, means of communication, and notions of context" (p. 5).

We must provide a mathematically literate workforce. Employees must be prepared to understand the complexities and technologies of communication, to ask questions, to assimilate unfamiliar information, and to work cooperatively in teams. The fastest growing fields are those that require the most education. The issue of opportunity is closely related to a good mathematics education. "Current statistics indicate that those who study mathematics are most often white males. Women and most minorities study less mathematics and are seriously underrepresented in careers using science and technology" (p. 3). Currently, mathematics educators and curriculum planners are responding to the challenge of providing a quality mathematics education for all students.

Natural Mathematical Knowledge

The studies of Saxe (1991) provide a look at the bearing cultural and social processes have on an individual's practice-linked mathematics understanding. The Oksapmin, a cultural group in a remote region of Papua New Guinea, adapted their number system, which is based on body parts, from enumerative to additive when the need for economic exchange became a goal of the group members. Brazilian candy sellers "showed the ability to solve complex arithmetical problems with very large values and ratio comparison problems" (p. 14). The motivation was economic survival but the outcome was mathematical understanding. Saxe compared the candy-selling children to their non-selling peers and documented a transfer of mathematical understanding to classroom type assignments.

As a result of these observations, Saxe developed a classroom practice in which mathematics was not the target of instruction. The instruction occurred in the "context of problem solving and took the form of assistance" (p. 18). The student would assume the role of treasure hunter and search for gold doubloons on a game board. Communication became critical since "the children were asked to use one another as opposed to a teacher as resources in solving problems with which they were having difficulty" (p. 21). The children gained mathematical competencies through the playing of a game.

One component of the practice sessions was the social environment. The concept of cooperative learning is not a new one. The language of mathematics and problem solving is definitely manifested in cooperative learning, but this area of research is extensive and is therefore beyond the scope of this paper.

Project 2061

Mathematics: Report of the Project 2061 Phase I Mathematics Panel (1989) identifies the language of mathematics as causing much of the difficulty that students now encounter in mathematics education. The project's authors identify mathematical language as "the careful use of natural language, clarified by certain conventions that eliminated ambiguity, and supplemented by the use of variables and carefully defined terms. These features of mathematical language enable mathematicians to formulate their concepts with utmost precision and to communicate propositions and their proofs in a mode that carries complete conviction" (p. 33).

The use of the conditional connective "if...then" in a manner of logical truth tables and the understanding of the connective "or" as inclusive are examples of conventions common to mathematical language. Mathematicians are also careful about the order in which negation and generality are employed.

The panel concludes that "it may well be that the true potential for mathematics to strengthen general problem-solving abilities lies in the nature of mathematical language" (p. 36).

Geometry

Fuys, Geddes and Tischler wrote a monograph (1988) that investigates how adolescents learn geometry in light of the Van Hiele Model. The van Hieles distinguish five levels of thinking that the learner passes though while learning geometry. Each level has its own language, set of symbols and network of relations. "Language structure is a critical factor in the movement through the van Hiele levels" (p. 7).

Van Hiele attributes many failures in geometry instruction to a "language barrier--the teacher using the language of a higher level than is understood by the student" (p. 7). The student accepts the explanation but the subject doesn't sink in. The student progresses from one level to the next by passing through the five phases: information, guided orientation, explicitation, free orientation, and integration. In the explicitation phase "the student becomes conscious of the relations, tries to express them in words, and learns technical language which accompanies the subject matter" (p. 7). The implication for instruction involves development of the student's informal language.

Thought and Language

Zepp (1989) identifies language as a key issue in mathematics education. Teachers that separate the two imply that thoughts and concepts exist independently from language. Zepp focuses on the relationship between thought and language and notices that comprehension of a mathematical statement requires a different way of thinking. This foreignness of thought is what "makes people regard mathematics as a foreign language and not a subset of whatever language it appears to be written in" (p. 4). Zepp points out that thought and language are not the same but "abstract mathematical thought cannot exist in isolation from language" (p. 4).

Linguistics

The study of language during the 1960's was dominated by Noam Chomsky. He viewed language as having both surface structure and deep structure. The surface structure corresponds to the sound and the deep structure to the meaning. Symbolic mathematics can be regarded as a language. For example, equations and inequalities certainly possess both surface and deep structure. In fact, mathematical equations with the same solution set can have different surface structure yet the same deep structure. By observing certain properties, mathematicians transform equations just as linguists follow the rules of grammar to transform English language sentences. Thus "Chomsky is saying that language can be studied in much the same way as mathematics" (Zepp, 1989, p. 6).

Linguists refer to the social context of a language as a register. Zepp identifies mathematics as a register. The most common mark of a register is its unique vocabulary. Clearly a group in the mathematical sense is different from a discussion group or a group of investors. "It may be this specialization of vocabulary that causes problems for students. They may confuse the everyday meaning of a word with its meaning in the register" (Zepp, 1989, p. 11). The other difficulty presented by the mathematics register is that it forces the child to remove the link to context.

Spoken language is communicative in nature and is full of references external to the words themselves. Voice intonation and gesturing are acceptable. Written language needs to be self-contained and able to stand on its own. Formal written language is precise and is perceived to be the hallmark of mathematical activity (Pimm, 1989).

<u>Summary</u>

Mathematics is a language or register within the native language of the user. The role of the educator is to be able to effectively and efficiently guide the student in its use through modeling and practice. Understanding the nature of language and how mathematics and language are similar will benefit the mathematics instructor, curriculum planner and most of all the student.

CHAPTER II

MATHEMATICS AS A SYMBOLIC LANGUAGE

Introduction

Similarities exist between natural language and mathematics in its notation, symbolism, and structure. In a language we can distinguish elements and actions. The nouns and verbs of natural language have as counterparts numbers and operations. Elements can be combined following existing rules to form coherent "sentences." There are an infinite number of sentences just as there are an infinite number of equations (Sinclair, 1984).

Use of Symbolism

The symbolism and notation found in mathematics was introduced to facilitate communication between people. Symbols were developed much like words are, out of need. Many students fail to acquire the conceptual meanings and functional purpose of the notation and symbols (Fagan and Thompson, 1989). In fact the inability to comprehend the symbolism and notation is a cause of math anxiety (Tobias, 1975).

The need for symbol use is questioned by MacKernan (1982). Could it be that words, rather than symbols, should be written in a mathematical expression? Mathematicians are opposed to such thinking, but what about the learner? For the symbolism to be meaningful one must be able to translate its contents into an informal oral form and then into a formal written form. MacKernan states that "a case can be made for a preferential use of words on certain occasions, at least in the teaching of mathematics" (p. 27). Since historically mathematicians progressed from words to symbols, the path of the individual appears to be much the same.

How easy is it for students to make a "translation" from natural language to mathematical notation? Looking at how younger children are taught arithmetic as compared to written language provides valuable information. Hermania Sinclair (1984) makes the following comparison between learning to read and learning arithmetic.

> The assumption made for reading, that is, that the child knows how to talk and merely has to learn to put speech down on paper, has no counterpart in mathematics. Nobody seems to think that children already know how to add, subtract, multiply and divide before they come to school, and that all they have to learn is to do pencil-and-paper sums. On the contrary-in most countries arithmetic is taught as if the conceptualization of arithmetic operations were the same as their written symbolization. Schools do not seem to envisage that the conceptualization of addition, subtraction, etc., may be a cognitive task separate from that of writing equations, and that the latter may present difficulties of its own. (p. 9)

Symbol vs Concept

This brings up a point that is central to the student's formal mathematics education. Being able to use the notation system for arithmetic is much different from understanding it. In a study on children's spontaneous use of symbolism (Sastre & Moreno cited in Sinclair, 1984), students that were capable of doing sums in school were asked questions about quantities and given paper and pencil to make notes. The students did not use numbers or plus and minus signs to illustrate the verbal problems. Students went through a process of inventing their own notation system using tallies, pictures of a hand, and other symbols. In their explanations they said words like "add" or "take away." They also indicated these operations on paper by crossing out portions of their drawings or crossing out a portion of the tallies they had drawn. "Using numbers and plus and minus signs to symbolize actions with objects certainly did not seem 'natural' to them" (p. 11).

Vygotsky (1986) validates the actions of the children as natural. The symbol is a second order stimulus. This means that it is self-generated and possesses "reverse action." The symbol operates on the individual, not the environment. If the students are to internalize the notation they must be the ones to give it meaning.

Piaget (Copeland, 1970) points out that the average child does not reach the conservation of number stage until age six to seven. Thus many of the symbolic activities of first grade should be delayed. Piaget explains that the concept of addition is understood if it is seen by the child as a reversible operation. For example, when the child sees 2 + 3 = 5 also as 5 = 3 + 2 =

1 + 4 = 2 + 3 = 4 + 1, they understand the concept of addition.

In the introduction to the teachers manual of the pre-K - 2nd grade series Mathematics Their Way (1976), Mary Baratta-Lorton explains;

> A page of abstract symbols, no matter how carefully designed or simplified, because of its very nature, cannot involve the child's senses the way real materials can. Symbols are not the concept, they are only a representation of the concept, and as such are abstractions describing something which is not visible to the child. Real materials, on the other hand, can be manipulated to illustrate the concept concretely, and can be experienced visually by the child (p. xiv).

Baratta-Lorton emphasizes a major goal of mathematics education as concept development and suggests that abstract symbolism tends to interfere with the understanding of the concept. Symbolism is used, but only to label a concept that a child already grasps.

Jerome Bruner took the developmental stages of Piaget and formulated three instructional stages that one observes when teaching a topic: enactive, iconic, and symbolic. Intuitive learning is done during the enactive stage. The child can have knowledge that is represented in the form of an action, like throwing a ball. This knowledge is not at the verbal level. Iconic knowledge has visual or perceptual organization. The child at this level can mentally manipulate the images of concrete objects. Work at the symbolic level is seen when the child can manipulate symbols that represent knowledge. An example would be mathematical calculations where the symbols are numerals.

Hermania Sinclair (1984) states that:

> It seems to me that young children can only learn arithmetic if they can attach meaning to numerals and equations. Arithmetic, like reading and writing, has to do with the extraction and construction of meanings-at least for children. The difficulty lies in deciding what meaning equations can have for young children. A simple translation into words is no help. From all we know about children as constructors of knowledge, mathematical meanings are constructed as action-patterns, first on real objects and later internalized. However, much research and much careful observation is still necessary on this last point (p. 13).

Necessary Language

Polya (1945) also supports the view of mathematical notation as a sort of language; "a language well adapted to its purpose, concise and precise, with rules which, unlike the rules of ordinary grammar, suffer no exception" (p. 135). The setting up of an equation is seen as a translation from ordinary language to the language of mathematical symbols. In ordinary language, some words have meanings that are dependent on context. Similarly, in mathematics, variables assume different meanings in different problems.

Polya points out an important step in problem solving to be that of choosing appropriate notation. Implicit in this approach is that the notation is not the mathematics, the notation is used to "do" mathematics.

Another similarity that Polya draws is that of "second meaning." When writing, we choose words whose meaning we want and whose second meaning doesn't detract from the use of the word. In mathematics the use of certain letters can provide trouble. Examples would be the letters e and i. By common use these stand for the basis of the natural logarithms and SQRT(-1), respectively. It would be safer to reserve such symbols as these for times when their traditional meaning is needed.

Polya's views of notation challenge the educator with the responsibility of helping the student experience the need for symbolism. The student must be "given ample opportunity to convince himself by his own experience that the language of mathematical symbols assists the mind" (p. 141).

Algebra

Inherent to any language are both grammar and meaning -syntactic and semantic components. Martha Burton (1988) identifies algebra as a symbolic language. "The power of language is not in the words themselves, but in the use that we make of them to communicate with each other. The words of our language support communication because they are symbols pointing beyond themselves to things we experience in our world. To be real, language has to be about something" (p. 4).

Algebra for many is a semantic-free language, void of meaning. "The student is unable to encode meaning from natural language word problems into algebraic symbolic language. And they seem not to be able to recognize meaning in an algebraic sentence either" (Burton, 1988, p. 4).

English sentences that can become algebraic statements are those that have to do with quantities. The algebraic nouns are quantifiable entities that appear as verbs in English. "Corresponding to each of these English verbs is a measure function *cost-of* and *weight-of*, so that sentences 'The coat costs $225' and 'The dog weighs 74 pounds' will be rewritten 'cost-of (coat) is 225' and 'weight-of (dog) is 74" (Burton, 1988, p. 5).

When constructing the algebraic sentence we have basically two verb choices "is" and "exceeds." Any mathematical "sentence" can be written with these, or as a combination of these two. The result of translating an English sentence to an algebraic sentence is an equation that summarizes the desired word problem (Burton, 1988).

Algebra can be compared to American Sign Language. American Sign Language is a dense language which consists of a set of simple symbols. Each symbol has multiple or complex meanings. In algebra, the fact that symbols have multiple uses can cause confusion. The "-" symbol has four different uses (Subtraction, opposite, negative, and negative exponent). "Learning based on algebraic words and phrases such as 'opposite' can be acquired faster and last longer when compared to learning based on the respective symbols '-(-4)'" (Rotman, 1990, p. 50).

Bruner

Jerome Bruner also addresses the issue of notation. "The *notation theorem* states that early constructions or representations can be made cognitively simpler and can be better understood by students if they contain notation which is appropriate for the students' levels of mental development" (Bell, 1978, p. 143). Efficient notation systems in mathematics make the extension of principles and the creation of new principles possible.

Bruner advocates a sequential approach to learning. Spiral teaching and learning is an approach where each mathematical idea is introduced in an intuitive manner and is represented using familiar and concrete notational forms. As the student matures intellectually, the same concepts are studied at a more abstract level and with less familiar notational forms.

With careful planning, many problems with notation can be avoided. An algebra student who just learned that parentheses are used for grouping will not be ready to use the notation $y = f(x)$ to represent the concept of a mathematical function. The concept of function can be introduced with the representation $y = 3x + 5$. In advanced algebra the student will be able to adapt to the $y = f(x)$ representation.

Summary

Symbols are not the concepts but in mathematics, symbols are necessary. Mathematics educators must first teach the concepts which create the need for the symbols. Since children go through the process of developing their own notation systems, this development should be part of their mathematics experience. Work with manipulatives, group activities with opportunities for verbalization, drawing, chart making, and problem solving can all lead to concept development. Once the concept is developed the educator can introduce the symbolism and the mathematics can be explored at a greater level of abstraction.

CHAPTER III

LANGUAGE DEVELOPMENT AND MATHEMATICS

Introduction

Mathematics is described as an activity involving a way of thinking and a means of using relationships to solve problems. "Mathematics is also a language - a means of expressing certain truths by the use of certain words and symbols. The truths are more important than the words and symbols which are used to express them, and it is essential that children should discover these truths if they are to understand and use the language of mathematics. To teach the language first, before there is knowledge and understanding, is comparable to teaching children to speak and read Latin without giving the meaning of any of the words" (Inder, 1982, p. 39).

Learning Language

With this is mind, Inder (1982) looks at the learning process in children. If children are given opportunities to explore patterns and make discoveries they will naturally begin to discuss their actions and the results with someone close to their level of thinking. Their use of natural language to justify their results, reinforces the learning process. The introduction of words and symbols that meaningfully describe their actions and results would be a logical progression for meaningful instruction.

The involvement of language calls for an investigation of how a child learns language. Inder applies the sequence of listening, speaking, reading and writing from language instruction to mathematics instruction. This provides the following sequence of events to provide meaningful learning:

- the activity
- the description
- the recording of symbols
- the use in problems and other situations. (p. 38)

The student that follows this sequence learns:

- What I think, I can say
- What I say, I can write
- What I write, I can read
- What I read, I can use. (p. 38)

This runs counter to traditional instruction where children are asked to interpret and manipulate symbols and sentences before the meanings have been made clear. The goal of instruction is that the student will see the deep structure of a problem and not just the surface structure. When faced with a multiplication problem like 1/2 x 4/5 the student can simply read it as "one half of four fifths." This is obviously "two fifths." Typically, this is not done. Students tend to jump in and apply an algorithm, or ask "What should I <u>do</u>?", without ever understanding the problem. They never see the deep structure of the problem. Older students that are used to this type of mathematics environment will even balk at the attempt from a teacher to explain the problem. The student wants to know how to get the answer and nothing more.

Similarities

There are many similarities between learning language and learning arithmetic. Hickerson (1959) outlines six areas.

Learning Language

1. Engaging in first-hand multi-sensory experiences. (Learning to develop a perceptual content of the biological and physical worlds.)

2. Listening to spoken word-symbols, singly and in sentences, which represent the things, ideas, and events experienced. (Learning vocabulary and sentence structure.)

3. Representing things, ideas, and events through oral language symbols. (Learning to express self and relate experiences orally.)

4. Identifying written language-symbols and relating them to spoken language-symbols and to first-hand experiences. (Learning to read with meaning.)

5. Representing things, ideas, and events with written language-symbols. (Learning to express self and relate experiences in writing.)

6. Acquiring knowledge and understanding of the logic of the language structure. (Learning grammatical definitions, rules, principles, generalizations; e.g., parts of speech and syntax.)

Learning Arithmetic

1. Engaging in first-hand multi-sensory experiences. (Learning to develop a perceptual content of the quantitative aspects and quantitative problem-situations of the biological and physical worlds.)

2. Listening to spoken word-symbols, singly and in sentences, which represent the quantitative aspects, quantitative relationships, or quantitative problem-situations found in the things and events experienced. (Learning the vocabulary and sentence structure used in describing things and what is happening to things.)

3. Representing quantitative aspects, relationships, and problem-situations orally. (Learning to express orally in sentences the quantitative situation, learning to compute orally, and to solve problem situations orally.)

4. Identifying written arithmetic-symbols and relating them to spoken word-symbols and to first-hand quantitative experiences. (Learning to read arithmetical symbols with meaning.)

5. Representing quantitative aspects, relationships, and problem situations with written arithmetic-symbols. (Learning to write numbers and operational signs in arithmetic expressions which represent quantitative situations and leaning to write algorisms in computation.)

6. Acquiring knowledge and understanding of the logic of the number system. (Learning mathematical definitions, rules, principles, generalizations; e.g., notation, place value, laws of association, distribution, and commutation.) (p. 242)

Language Ability

Is there a relationship between a student's language abilities and the learning of mathematics? Hamrick (1976) investigates oral factors in readiness for written symbolization of addition and subtraction in first graders. The study is based on "the assumption that written mathematical symbols are similar to written language symbols. When a student is learning either type of symbol, he must associate the symbol to a meaning" (p. ii).

Hamrick compares the learning of the written symbolization of mathematics to learning the symbolization of written language, or learning to read. "In reading and language education, verbal facility is considered to be an important readiness factor; a child is not considered ready to read until he has an adequate speaking and hearing knowledge of the words and sentences he is expected to read" (p. 2). Traditionally, little consideration is given to the spoken vocabulary in relation to the symbolic mathematics that a child must learn in the mathematics setting. In fact many children are introduced to mathematical concepts through the symbols.

In the study, students were first classified by Hamrick as either ready or not ready. Readiness for the symbolization of a topic depended on the students ability to master objectives of the topic verbally and perhaps with the aid of pictures or manipulatives. Hamrick concludes that "if a student is not ready for the introduction of symbolization of addition and subtraction, the student's learning of the symbolization will be more meaningful and more efficient if the symbolization is delayed until the student is ready" (p. 86). This study highlights the importance of verbal activities prior to written activities at the first grade level.

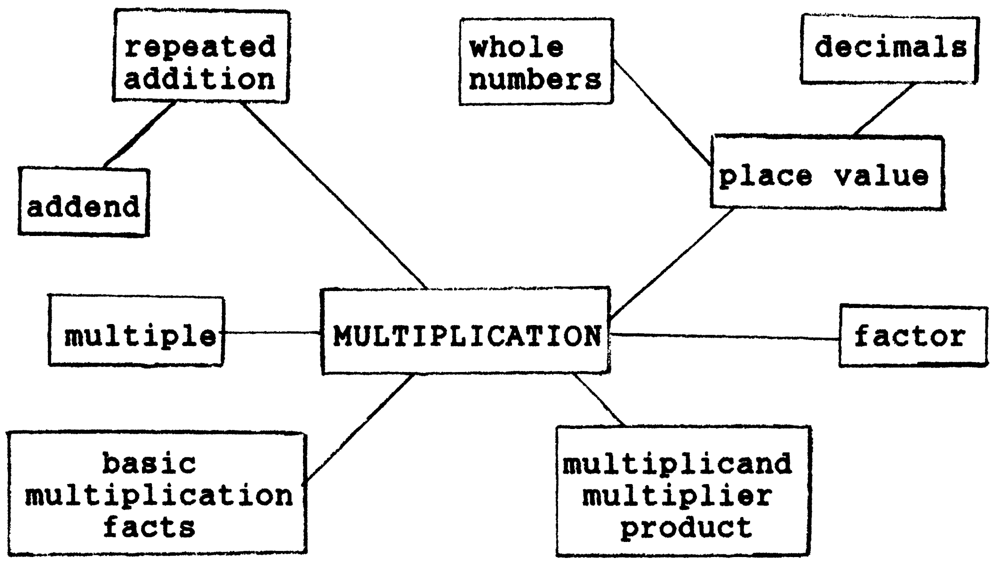
Language facility is also found to be important for junior high students. Bradley (1990) studied the relationship among students' mathematics language facility, procedural mathematics knowledge and understanding, and applications of mathematics concepts. Bradley found that for the average and the above-average students, mathematics language facility significantly correlated with both procedural and conceptual knowledge. Also, language facility and procedural knowledge together were found to be a powerful predictor of conceptual achievement.

Bradley concludes that these findings have strong implications for mathematical instruction. "Mathematics teachers should incorporate mathematics language development into current teaching practices" (p. 26).

Vocabulary

Central to language development is development of the vocabulary. In order to be an effective problem solver the student must comprehend the verbally expressed problem. Heinrichs and Larrabee (1989) outline instructional methods that move the student from their everyday language into the language of mathematics. In the first activity, students working in groups make a two-dimensional map of the relationships between mathematical words. Figure 1 shows an example of the semantic map for the term multiplication.

Figure 1. Semantic Map of Multiplication Related Terms.



Note. From "Teaching the language of mathematics in the upper elementary grades" by A. S. Heinrichs and V. Larrabee, 1989. In G. W. Blume, & M. K. Heid, (Eds.), New Directions for Mathematics Instruction 1989 Yearbook p. 48). University Park, PA: Pennsylvania Council of Teachers of Mathematics (ERIC Document Reproduction Service No. ED 309 989).

The students gain a sense of power over the concepts as they discuss which terms and paths of relationships might be chosen. A second activity addresses the fact that many words have additional meanings that are in common use. Students are given a list of words and are asked to write a story using the mathematical words in their alternate meanings.

An additional activity that deals with multiple-meaning words is to have the student create several sentences that illustrate the various meanings of the word. The student becomes aware of the multiple meanings of words and the importance of exact definition in the use of technical vocabulary. Increasing the students' vocabulary will in turn increase their comprehension of oral and written work.

Word analysis is a linguistic approach that studies the meanings of words by analyzing the meanings of structural elements of the words. This approach reveals that most mathematical terms have prefixes, suffixes, and roots that offer clues as to the terms' meanings. Milligan & Milligan (1983) suggest that students make up vocabulary cards for math terms. "On the card they define the word, identify its elements, and list other words that contain the same elements" (p. 489). An advantage to this activity is that it equips the student with the skills that can assist them in determining the meaning of many unfamiliar words.

The problem of vocabulary seems to be magnified for students who have learned English as a second language. Garbe (1985) found a significant difference in the conceptualization of mathematical terms by Navajo students and their Anglo peers. One difficulty was with terms that sound like other commonly used words. An example of a term that caused confusion is "sum," since it sounds like the words "some" and "sun." When asked to choose the meaning of the word "sum," Navajo students answered "a part of something" at a rate of 28% and later in the test selected a drawing of a sun, at a rate of 20%. One explanation for the error is that the Navajo students' function in two languages and the mathematical terms had not been effectively distinguished from "sound alike" words.

Garbe also concluded from his study that reading ability is a greater factor than mathematical ability in the conceptualization differences exhibited by the students. Garbe points out that "teaching vocabulary is not teaching mathematics, but is one of the skills that must be taught in mathematics" (p. 42).

Summary

Language development includes the following sequence of activities; listening, speaking, reading, and writing. Since mathematics is a language, educators will benefit from incorporating the sequence of language development into their instruction. Specific attention can be given to vocabulary and vocabulary-building exercises. The mathematics educator needs to provide an environment where mathematics is spoken. For the learning to be genuine we must move away from lectures and worksheets and enter into mathematical discussion with our students. The following chapters will further develop the topics of verbalization and writing in mathematics education.

Chapter IV

VERBALIZATION AND MATHEMATICS

Introduction

What role does verbalization have in the mathematics classroom? Greeno (1988) studied students' concepts of function with the aid of a function machine. In the study students were asked to talk about the machine. There were many occasions in which the meanings of words had to be negotiated between the pairs of students being interviewed or with the interviewer. Greeno noted that the students implicitly understood the concept in question (function) but had to work to express it in language.

In fact some students could not tell how they figured something out. When questioned, some students would offer little more than "I just know it." In this situation the student may not have the "words to tell anyone what mental processes led to a particular conclusion" (Lampert, 1988, p. 468).

Unverbalized Awareness

Gertrude Hendrix (1988) identifies this as "unverbalized awareness." This stage is reached after a student has made a generalization but before attempts are made at construction of a verbal or written form of the generalization (which is labeled "conscious generalization"). The study done by Hendrix compares the work of three groups. The first group learned by authority. The generalization was given and students applied the generalization to complete a sample of exercises. The other two groups learned through an inductive-deductive approach. The groups of students were given a problem that they could not answer quickly. The instructor modeled problem-solving strategies to aid the students in discovering a rule for the given problem. Students in the first of these two groups were not asked to write down the rule and were therefore in a stage of unverbalized awareness. The other group was asked to compose a sentence which states the generalization just used. Out of 42 subjects only two successfully communicated the rule on their first attempt.

After the experimental sessions, the subjects were given a test in which all answers could be found by counting. Some of the test items could also be found quickly if the generalization learned in the experiment session was used. The results of the study showed that the conscious generalization group did twice as well as the group that learned by authority. However, the unverbalized awareness group did even better than the conscious generalization group.

Hendrix maintains that students may not have the language necessary to state the generalization correctly. Once the language is in place they can proceed. Another factor may be the sense of finality they feel once the rule has been stated. The student is "done" and pays less attention to it. Another theory is that the generalization is lost to the students if they become confused over the literal interpretation of an incorrect statement. A student may take the incorrect statement to be the new rule.

The fact that researchers have validated this stage of unverbalized awareness runs counter to the belief that "if you can't say it, you don't know it."

Difficulties are also cited by Schoen (1984) as stemming from two different sources. First, the student has difficulty translating from number language to word language. Second, they are struggling to change levels of abstraction. The student can visualize a specific case but cannot make a generalization that would include every situation.

Schoen made observations while teaching both entry-level mathematics and entry-level English programs. She compares the transition in and out of the number language as being similar to the transition made by one who speaks a second language. Schoen states that "it is an area of limbo, of nebulous thought process and unclarified syntax. It is here that one gets one's thoughts straight--if one can. The transition language is uncomfortable because it is not recognized and certainly not accepted" (p. 12). What Schoen suggests is that we aid the student by letting them address a problem in their own natural language. She identifies the language used by students as "student-friendly." Schoen compares the use of natural language to the prewriting process that students go through when preparing an English paper. The core of the idea is what is important and is developed first, while the organization and syntax soon follow.

Piaget

The role of language is central in the learning theory of Piaget. Piaget was criticized in his early work "because he drew conclusions from children's answers at the verbal level" (Copeland, 1970, p. 9). After further study, Piaget believed that language often indicates a child's stage of development but cautioned that "words are probably not a short cut to understanding; the level of understanding seems to modify the language that is used rather than vice-versa" (cited in Copeland, 1970, p. 13).

Piaget (Copeland, 1970) was also interested in the fact that children could not tell how they got an answer. He attributes this to the child's inability to analyze his own reasoning. The process of verification of an answer involves logical thought. The child operates in a world of perceptual knowledge but not one of logical rational knowledge. Central to logical thought is language. Lowenthal (1984) observed that children who work in groups acquire a logic through their language and vice-versa.

Mathematical Knowing

Greeno (1988) discusses the role of language in learning;

> In teaching mathematics we often begin by giving students definitions of terms, and then we expect them to use the terms correctly because "they should know what the words mean." This is probably a wrong-headed way to think about language in learning. Formal definitions have an important role in mathematics, of course, but they are not the main ways in which terms acquire meaning for communication. Instead, we need to create situations for students to communicate about with each other and with their teachers. The process of communication is an important vehicle for developing more articulate forms of understanding, and may be the main cognitive resource for developing general forms of knowing. (p. 495)

Mathematical "knowing" has two different connotations. Knowing mathematics in school means having a set of unexamined beliefs. The problem is correct because either the teacher or the textbook says it is. "Lakatos and Polya suggest that the knower of mathematics needs to be able to stand back from his or her own knowledge, evaluate its antecedent assumptions, argue about the foundations of its legitimacy, and be willing to have others do the same" (Lampert, 1988, p. 437). The situation where the student can practice using the tools of language and symbolism are central to mathematical "knowing."

An instructional approach which includes induction and deduction may prove useful. As Lampert (1988) states, "the problem is not the question and the answer is not the solution" (p. 444). The student's main achievement in the solution of a problem is to conceive the idea of a plan. The strategies used for figuring out how to get this plan, and the ability to reach a solution, constitute mathematical knowing. Integral to Lampert's theory is the "interaction of the words 'knowing,' 'revising,' 'thinking,' 'explaining,' 'problem,' and 'answer'" (p. 442).

Lampert's use of "mathematical discourse" involves a major shift of the roles of student and teacher. Students solve problems by proceeding through a process of guessing and revision until an assumption is validated or a counterexample is found. The result, claims Lampert, is "that the students had learned to regard themselves as a mathematical community of discourse, capable of ascertaining the legitimacy of any member's assertions using mathematical form of argument" (p. 447).

Mathematical discussion is defined by Pirie and Schwarzenberger (1988) as talk that has the following properties. "It is purposeful talk on a mathematical subject in which there are genuine pupil contributions and interaction" (p. 460). Communicating mathematically is one of the NCTM's six major goals for mathematics education, yet what takes place in most classrooms is teacher-led talk, not mathematical discussion.

Schoenfield (1983) states that we are not teaching students to think but providing them with "thinking skills that they can use after they take our final exams" (p. 7).

The instructional method that Schoenfield advocates is one of cognitive apprenticeship (Collins, Brown and Holum, 1991). The six teaching methods promoted are: modeling, coaching, scaffolding, articulation, reflection and exploration. Modeling involves the teacher solving a problem while making the cognitive process obvious to the student. This involves the use of heuristic strategies as the teacher makes comments like, "Can I solve an easier problem?" The students learn the problem-solving process which is necessary for them to articulate their understanding of concepts and procedures. The teacher's role of "expert" is transformed as the goal of cognitive apprenticeship is that the student becomes the "expert."

<u>Summary</u>

The role of verbalization is an important one. Through verbalization the students communicate their ideas. The effect verbalization has on "conscious generalization" is somewhat controversial, but many would agree that a student may "know" something and not be able to express how they have that knowledge. The student may not have the vocabulary or the ability to examine their own thought process. The instructor needs to be sensitive to differences in students' learning styles. One learner may be able to verbalize easily while another may struggle and even lose ground when forced to verbalize. The educator needs to provide an environment where students are "speaking math". Only through active participation will mathematical thinking take place.

Central to the issue of verbalization is what is said by the mathematics educator. Educators most often ask the student, "What is the answer to . . . ?" The educator could model their own thought process by asking, out loud, "What steps do I use to work this problem?". Modeling the heuristics of problem solving is the most effective method of transmitting them to the student.

CHAPTER V

WRITING AND MATHEMATICS

Introduction

One goal of the mathematics educator is to move the student from the predominantly informal spoken language to the formal written language. David Pimm (1989) outlines two strategies. "The first is to encourage students to write down their informal utterances and then work on making the written language more self-sufficient ... a second route might be to work on the formality and self-sufficiency of the spoken language prior to its being written" (p. 65).

Why write in the mathematics classroom? How can we encourage students to think? One clue comes from cognitive research. Studies indicate that thinking, speaking, reading, listening, and writing are interrelated: One reinforces another as students construct knowledge. Each is also an opportunity for reflection. Written assignments in mathematics classes also afford students the opportunity to organize their thoughts and at the same time, improve their writing skills. If students can write clearly about mathematical concepts, then it is apparent that they understand them (Johnson, 1983).

Many forms of writing have a place in the mathematics classroom. Logs, journals, expository writing, and creative writing can be used effectively (McIntosh, 1991). Long range assignments can include preparing a manual for other students, revising a technical manual, and writing about famous mathematicians. Students can produce a mathematics newspaper or write short poems or rap verses about such topics as slope, limits, rectangles and so forth (Mason, 1991). Writing tasks also provide an opportunity for cooperative activities between mathematics and other disciplines.

Journals

Journals can assume a variety of forms depending on the teacher's purpose. Journal writing is effective in opening the lines of communication and helps build a sense of trust so that students can take risks (McIntosh, 1991, and Schmidt, 1985). Nahrgang & Petersen (1986) state that the journal "offers students the opportunity to work informally and personally on mathematical concepts, using their own language and real-world experiences . . . The journal goes beyond rote learning and challenges the student to use intellectual skills" (p. 461).

The journal is like a diary. Each entry is a short written response to an instructor's question, statement or set of instructions. All responses are written in prose rather than in the traditional mathematical style. According to Nahrgang and Petersen, the less math in the student's writing, the greater the understanding. A journal can be evaluated, but studies indicate that grading journals weakened their effectiveness as a learning tool (Nahrgang & Petersen, 1986).

In education, emphasis is usually placed on the cognitive domain. This is especially true in the mathematics classroom. The student that does not have success in the classroom may be suffering from math anxiety or experience mental block on tests. Writing is a personal way in which the instructor can get feedback from students that would not speak up in class. The degree to which writing is successful is determined by the response given by the teacher. Personal and encouraging remarks by Watson (1980) started a two-way conversation that was beneficial to the class. Her students realized that she cared about them and looked inside themselves to solve their own mathematical problems. The result was improved grades for many students.

Communication

Writing gives students another way to look at math problems. Mathematics is, after all, communication, but communication in math involves a compact, unambiguous symbolism that to many students is cold and rigid. Writing, on the other hand, is a less structured way of expressing ideas (Schmidt, 1985).

Esbenshade (1983) builds a geometry unit around the popular British novel Flatland with the goal of humanizing math education. "Including Flatland in the curriculum may help with what Guting refers to as *preparation for life* and what Wheeler terms the *expansion of human awareness*, especially that of a mathematical nature" (p. 122).

A common saying among teachers is, "we never really understood something until we had to teach it." Psychological theory indicates that verbalization at the appropriate time improves our ability to recall and organize information. Questioning is a means to help students verbalize their thoughts and give them feedback.

Written explanations of mathematical concepts have several advantages over discussion. All students can participate simultaneously. Teachers can consider written responses more carefully than verbal ones. Writing encourages more precise work thus increasing the students' technical writing skills. The teacher and student can review the work together and discuss specific problems (Geeslin, 1977).

Where do we find time to add writing to an already full curriculum? At the college level, writing can replace quizzes (Nahrgang & Petersen, 1979). Students can be given a writing exercise at the beginning of the class when the teacher is busy with routine activities or after completing a test (Watson, 1980).

<u>Vocabulary</u>

Often confusion stems from the same terminology being carried over from arithmetic to algebra. The multiplication operation is written symbolically, and that makes it difficult for students to see the relationship between a factor in arithmetic and a factor in algebra. Before teaching the factoring technique, the student needs to know the concept of factor and how it applies to algebra. The text gives examples and the instructor usually repeats them on the board, but this is often not enough. After the instructor defines a term and its application to algebra, it may be beneficial to have the student define it in his own words and make up his own examples.

In a learning hierarchy as defined by Robert Gagne (Bell, 1978), concept learning precedes rule learning. Many students can memorize a definition and do not comprehend the concept. By asking students to "explain" a concept, a student exhibits an understanding of the given concept. The student must provide more than a memorized statement.

When asked to define a circle most students emphasize roundness, the measure of the central angle or some other nonessential feature. "Asking students to explain how to construct a circle with a string and a piece of chalk and then to explain how the points are related pushes them to refine their definitions. In addition, it exposes misconceptions that are due to over-emphasizing visual features rather than geometric properties" (Carroll, 1991, p. 19).

The role of the mathematics teacher is not to teach writing (Mett, 1987) but the experience will help improve the student's technical writing skills (Geeslin, 1977). Writing is not language, but writing is a form of language that can be used to facilitate instruction in mathematics education.

Observations

My experience with writing began as I observed the following situations. A student called factoring, "unfoiling." In his mind, factoring was the opposite of multiplying, using the "foil" technique. Another student was performing a canceling operation, but when asked what he was doing, he responded "crossing out." I realized that my students must be having trouble with the vocabulary I was using.

To get written feedback from my students I added a prompt to the daily quiz. The prompt required a short written response from the student and was not graded. For example, I asked the students to define an algebraic expression using only words. Table 1 lists the student responses. Many responded by telling what is not an expression. The students compared an expression to an equation and commented on the difference: no equal sign or that it could not be solved. These responses indicate a poorly developed concept of equation. Another prompt asked for the definition of the term trinomial. Table 2 lists the student responses. Again we see misuse of the word equation and the comparison of a polynomial to an equation. The responses, "product of two binomials" may be true for a specific trinomial but do not hold true in general. I have observed many students describe the terms of a trinomial as having a variable with the power decreasing from two to zero. This again is a specific case and not general enough for a definition.

Table 1. An algebraic expression is:

1. a group of terms used to express a mathematical symbol which can't be solved.
2. some terms with multiply, divide, add, or subtraction sign but no equals sign.
3. the answer that you get or problem that the final answer is not set equal to something. It just stands alone there is no x equal to something.
4. any equation that doesn't have an equal sign there is no answer.
5. equation without an = sign it can not be solved.
6. a group or term that may involve a function to solve it.
7. not a solution has no equal signs. Just simplified.
8. something that does not have an equal sign but it has numbers.
9. a polynomial (bi, tri, etc ..) that has no equal sign and cannot be solved just simplified.
10. a formula that doesn't equal anything.
11. a numerical statement that has no equal sign in it.
12. a mathematical statement not equal to zero.
13. a group of numbers that can not be solved. The problem contains no equal sign and has no answer. All you do is simplify or group like terms.
14. a mathematical term that can not be broken down to be solved for only 1 variable.
15. a math problem with no equal sign.
16. a group of numbers but do not equal something.

The results to these questions, and other similar prompts indicated the confusion that some students were experiencing with terminology. I then focused on vocabulary and used writing as a means of communication between myself and my algebra students. As we worked through algebra problems, we carefully noted the steps that had been taken. In some cases we discussed why an algorithm was applied. We often related an algebraic problem back to arithmetic, especially when working with rational algebraic expressions. It was obvious that many

Table 2. What is a trinomial?

1. A monomial with three elements in it.
2. The term trinomial means the equation is made up of 3 variables as in $x^2+2xy+4y^2$.
3. A trinomial has three different components in the expression.
4. Three terms, separated by a sign (subtraction, addition, divide, multiply).
5. Two (2) binomials multiplied.
6. A mathematical expression containing 3 numerical terms.
7. An equation which has three binomials.
8. Instead of 1 variable squared and 1 number there is 1 variable squared, a variable and a number.
9. An expression with 3 numbers or variables in it that can be factored.
10. The product of two binomials.
11. Trinomial has 3 factors.
12. Has 3 part monomial = trinomial.
13. It is an expression with three terms.
14. A trinomial is an equation or group of numbers that there is 3 of with a variable in 2 of them.
15. It is an expression with three number groupings being mathematically manipulated.
16. An expression w/three binomials - not equal to zero.
17. When there's three set of parenthesis - there's three groupings instead of two.
18. An expression with the highest exponent is 3.
19. An expression consisting of three separate terms.
20. A trinomial has three terms like this $3x^3+15y+56$.
21. A trinomial is a polynomial w/3 factors.
22. An expression with 3 different variables.

students lost sight of the fact that they were manipulating a fraction.

Summary

In language development writing logically follows speaking. Writing is similar to speech in the degree of formality involved. The process of writing is important in itself. As a person writes, thoughts are formalized. With practice the student is better able to express himself both orally and in written work.

Writing can also be used as a means of communicating. Older students may have math anxiety or a bad attitude caused by past failure. Writing can be effectively used to change a student's feelings about how they learn mathematics. Through journals I had students write to me about their experience in algebra. Students who experienced math anxiety and others who felt that they were failures in mathematics communicated these feelings to me through their journals. Once the students experienced success with the mathematics, their attitudes changed. It was a rewarding experience to watch them learn that they could do mathematics.

CHAPTER VI

TEACHING STRATEGIES AND IMPLICATIONS FOR MATHEMATICS INSTRUCTION

Introduction

This final chapter concentrates on a variety of teaching strategies that are based on a linguistic approach to the teaching and learning of mathematics. Writing and the verbalization of mathematical concepts can be incorporated into the learning environment and have been previously discussed. Attention is given here to solving story problems, vocabulary instruction and testing and symbolic notation. Personal observations of the author are also included.

Word Problems

While directing a Mathematics Center, Martha Burton (1988) observed many calculus students experiencing difficulty solving word problems. The difficulty was attributed to students' inability to translate the problem from natural language to algebra. Burton proposes a different approach to word-problem instruction. Traditionally, we teach following a known-unknown method. We make variable assignments and then link the expressions to form the algebraic equation. Often the process breaks down when the student fails to recognize how to set up the equation. Even more confusing for some, is a method where conditions are represented as quantities in a chart. Here, prior understanding of the problem is necessary to choose an appropriate chart. After observing my own students flounder at word problems, and hearing comments from capable students like "I can't see what I'm supposed to do in the word problems", I welcomed a new approach.

Burton suggests that the whole sentence that would be used to represent the problem be assembled in English first. This makes the verb "is" or "exceeds" available from the beginning. The student can then think about the pieces that are needed and assign variables as needed. For the word problems involving work that we were facing, we used the sentence, "The total rate of work done when working together is the sum of the individual rates for the job." This provided the framework for this type of problem. The idea of rate was then discussed. This was followed by the selection of a variable. After setting up the equation, the students were eager to solve it. They finally saw the purpose of the algebra that they had learned. The response to their success was the best part, as the same student verbalized, "I can do these now."

Vocabulary Instruction

To understand the benefits of vocabulary-oriented instruction in a mathematics class, Jackson and Phillips (1983) observed seventh grade students that were studying the topic of Ratio and Proportion. In the study, the control group of classes and the experimental group of classes both used the same lessons, activities, text books, materials, and procedures. The experimental group received five to ten minutes of vocabulary-oriented activities. A team of language experts, which included secondary school teachers, reading teachers, university professors of mathematics education, and mathematical language specialists identified the five symbols and six terms (Table 3) as essential for the proportion and ratio lessons.

Table 3. Essential Terms and Symbols Related to Ratio and Proportion

Term	Symbol
Approximation	$\doteq$
Centimeter	cm
Graph	
Meter	m
Proportion	$\frac{a}{b} = \frac{c}{d}$ or a:b=c:d
Ratio	$\frac{a}{b}$ or a:b

Note. From "Vocabulary Instruction in Ratio and Proportion" by M. B. Jackson and E. R. Phillips, 1983, _Journal for Research in Mathematics Education_, _14_, p. 338.

The vocabulary instruction used by the cooperating teachers involved the following components:

1. Recognize and identify terms and symbols.
2. Attach literal meaning to terms and symbols.

3. Categorize terms and symbols by inclusion and exclusion.

4. Identify examples and non-examples of concepts represented by terms or symbols. (p. 338)

Table 4 gives an example of the different types of vocabulary-oriented activities used by the cooperating teachers.

Table 4. Vocabulary-Oriented Activities

Type	Sample Activity	Answer
A	Working with your partner, decide whether you agree or disagree with each of these statements. 15:35=n:90 is read "35 into 15 equals 90 into n."	Disagree
B	With reference to chapter 13 of your text, decide whether each of the following is true or false. You must be able to prove your choice by reading from the text. Any comparison is a ratio.	False
C	Working in pairs, circle the term which *includes* all others. ratio comparison fraction phrase	Ratio
D	Which of these statements are true? Justify your decision. boy:man=girl:woman is a proportion.	False

Note. From "Vocabulary Instruction in Ratio and Proportion" by M. B. Jackson and E. R. Phillips, 1983, Journal for Research in Mathematics Education, 14, p. 339.

Jackson and Phillips conclude, "The data analysis clearly indicated that those students who received vocabulary-oriented instruction achieved higher verbal and computational scores than their control group counterparts at a statistically significant level, other effects being controlled" (p. 341). They noted that these classes received less time in class to practice computational skills since time was taken for vocabulary instruction. Jackson and Phillips also note that "the cooperating teachers indicated that they found the vocabulary-oriented activities easy to integrate into their normal instructional activities" (p. 341).

This type of study should be repeated with children of different ages and with larger samples. If the results are similar, the implications for mathematics educators is clear. Vocabulary instruction <u>will</u> benefit the mathematics student.

<u>Vocabulary Testing</u>

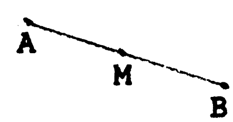
Fundamental to any language are words. Words make up the language and these words name fundamental concepts. Nicholson (1989) maintains that this is "certainly true of the language of mathematics. Hence, it is of great importance to diagnose whether or not key words are available to pupils and are properly understood" (p. 44).

After initial pilot testing, Nicholson tested a sample of almost 600 junior high students in Belfast, Ireland. Two different types of tests were used to evaluate the understanding or lack of understanding of particular mathematical words or terms. The first test, M1, asked straightforward vocabulary questions. In test M2 a mathematical statement was formulated which represented a particular concept, and the student was asked to fill in the blank with the name of the appropriate concept. Figure 2 shows sample questions from the vocabulary tests used in his study.

Figure 2. Sample Test Questions.

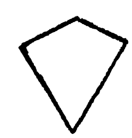
Questions (on Test M1)

5. Give one example of a multiple of 15 __________
6. Which of the following are integers? __________
   3, 1, 1/2, 0, -2, 2 1/4, $\sqrt{2}$
9. What is true about the sides of a parallelogram:
   ____
14. Draw a line from M perpendicular to AB:



Questions (on Test M2)

5. 3/5 is an example of a ___________ fraction.
6. The ____________ of 7 is 49.
7. A quadrilateral with one (and only one) axis of symmetry is called a ________________.



16. The numbers 1, 2, 3, 4, 5 (and so on) are called the ________________ numbers

Note. From "Mathematics and language (revisited)", by A. R. Nicholson, 1989, Mathematics in School, 18(2), p. 44.

Nicholson reported that "these tests proved useful to the teachers of the classes involved, as diagnostic instruments and the point was made that it is relatively simple to devise and administer similar short diagnostic tests in either of the alternative forms taken by M1 or M2" (p. 44). While evaluating the tests the teacher should take time to notice both terms that the student has mastered and those that need work. By noticing the errors that the student has made the teacher can devise a plan of remediation.

The results from the first test, M1, is reproduced in Figure 3. The eighteen mathematical terms from the test are presented in rank order of acceptable responses. The percent of students that gave an acceptable answer is also indicated. This type of diagnostic testing would be easy to do and the results could be used by both the student and the instructor to facilitate instruction in

Figure 3. Vocabulary Test M1 Results.

Mathematical Term	Percent
multiply	97.6
reflection	94.5
divide	94.3
factor	90.1
square root	87.7
parallel lines	85.4
prime number	79.9
angle (size)	79.2
volume	78.9
square number	78.0
quadrilateral	70.7
ratio	67.4
rectangle	64.5
axes of symmetry	54.2
perpendicular	39.5
multiple	27.4
parallelogram	27.3
integers	16.0

Note. From "Mathematics and language (revisited)", by A. R. Nicholson, 1989, Mathematics in School, 18(2), p. 44.

the language of mathematics.

Symbolic Notation

Many students think that mathematics is just the symbolic notation used by mathematicians. An activity that involves researching historical facts about mathematic notation and symbols (Fagan & Thompson, 1989) may help take some of the mystery out of mathematics. Students gather the following information about the symbol or notation in question: mathematician who first used it; nationality; year of birth/death; year of initial use and the source. By studying the historical evolution of symbolism the student sees that the symbols were created as an aid to communicating mathematics.

Translating equations from English to algebra is a skill that has caused confusion for many of my students. What is missing from the progression is "reading" an algebraic equation. No time is spent translating from algebra to English. It must be assumed by the authors of the textbooks that I have used, that students have this skill and do not need to learn it. Actually students are taught to solve equations and inequalities but not to read them, or even understand what they are. I propose that more time be spent on the development of the concepts of equation and inequality. Also, the student needs to be able to translate algebra to English in addition to translating English to algebra. The student will quickly see that an operation can be represented one way symbolically and many ways in the English language. Thus the symbolic language is more precise and universally understood.

Arithmetic and Language Teaching

Mathematics is similar to language. Therefore, instructional methods from the domain of language can effectively be carried over to the domain of mathematics. Hickerson (1959) outlines the implications for teaching both language and arithmetic.

Language Is Learned Best When

1. the learner engages in many varied first-hand multi-sensory experiences;

2. oral vocabulary and sentence structure are acquired in relation to the learner's experiences by listening to and talking about the things experienced;

3. written words are read as symbols standing for already known spoken words;

4. the written or spoken symbols have meaning to the reader or listener when they represent something perceived in his experience;

5. the writing of words and sentences is learned after the learner can read;

6. language usage is acquired in childhood gradually by imitation, experimentation, correction--not by memorizing grammatical rules and applying them.

7. grammatical rules, principles, and generalizations are taught by the inductive-deductive method, i.e., the learner is helped to educe and formulate rules, principles, and generalizations from past experience and then test them in subsequent experience.

8. there is a continuous interrelationship existing among widening and deepening first-hand

experiences, oral communication, written communication, and increasing consciousness and knowledge of the nature of language.

Arithmetic Is Learned Best When

1. the learner engages in many varied first-hand multi-sensory quantitative problem-situations;

2. oral language is acquired which represents in complete sentence form the quantitative relations in problem situations;

3. written arithmetic symbols are introduced as shorthand ways of writing already known spoken words;

4. the written or spoken arithmetic symbols have meaning to the reader or listener when they represent something perceived in his experience;

5. the writing of numbers; number combinations, algorisms, etc., is learned after the learner can read them;

6. computational processes are acquired gradually by manipulation of objects, imitation, experimentation, discovery, correction--not by memorizing mathematical rules and applying them.

7. mathematical rules, principles, and generalizations are taught by the inductive-deductive method, i.e., the learner is helped to educe and formulate rules, principles, and generalizations from past experience and then test them in subsequent experience.

8. there is a continuous interrelationship existing among widening and deepening first-hand experiences with the quantitative aspects of life, expression of these experiences with oral and written arithmetical symbolism, and increasing consciousness and knowledge of the nature of arithmetic. (p. 243)

Conclusions

In our country we have identified the problem of illiteracy, but an even greater portion of our society are math-illiterate. The factory-style of education which dictates that all high school freshmen will take Algebra has got to go. What is the answer? The National Council of Teachers of Mathematics has responded to the challenge as have mathematic educators around the world. Research for this paper reached to Canada, Europe, New Zealand and Asia. One element that is missing from the day to day, practical work of students is the recognition of mathematics as a language. NCTM has identified this and calls for "communication." What is becoming evident is that teaching mathematics does not mean teaching students to be calculators. We have the technology and need to be educating thinkers. The role of mathematics education is changing and the curriculum has to be responsive. Incorporating a linguistic approach is one change that is necessary.

Mathematics educators need to provide problem-solving situations and activities that will challenge the student. The sequence of language development will be evident in such an environment. The vocabulary of mathematics should be taught in the mathematics class. Where else is the student going to be exposed to this abstract, symbolic language? The student also needs to be given the opportunity to engage in meaningful mathematical discourse. Only through participation will the mathematics of the classroom come alive.

The issue of readiness is one that gets much attention in the primary grades. How do we know if a student is "ready" for the symbolism and abstraction of algebra? Currently the only prerequisite is to be a freshman in high school. If a student is not ready, why put them in a situation where they will fail? This is an issue that needs further research.

Textbook authors and publishers can aid the mathematics educator by providing textbooks and materials that incorporate language building exercises with the mathematics, as previously described. Concept development and vocabulary work needs to precede symbol use. Problems need to be presented in an open-ended fashion so less emphasis is placed on the answer and more emphasis is placed on getting the answer.

Finally, university professors that teach mathematics methods courses need to be aware of the research that links mathematics and language. They are the ones that can pass this information on to future teachers of mathematics.

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